*MECE 5397*

*Scientific Computing for Mechanical Engineers*

**Final Project**

**Solving the 2D Diffusion Equation using the Explicit and ADI Method**

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Report – Discretization of mathematical problem (10%) –

Description of algorithm (10%) –

Verification (15%) –

Presentation of results •

Grid independence (5%) •

Graphs and tables (10%)

**Introduction**

The Diffusion equation is a widely applicable partial differential equation. It was first used to solve problems related to heat transfer, but it can be used in many other ways. It has applications in other fields such as material science, information sciences, and life sciences. Though there are analytical solutions to the diffusion equation, in most real world applications they cannot be used. Numerical methods are used to solve diffusion equations without an analytical solution. Two methods are going to be examined in this study. The first method is the explicit discretization of the diffusion equation. This discretization can be solved using iterative methods. The second method is the implicit discretization of the diffusion equation. This method requires the use of the alternating direction implicit (ADI) scheme to solve.

**Problem Statement**

The equation presented that is being solved is

This is the 2D diffusion equation. The domain being examined in this paper is

And the boundary conditions given are

**Discretization**

The given diffusion equation consists of one first-order term and two second-order terms. The second order terms can be discretized using the standard second-order approximations

(x-direction approximation)

(y-direction approximation)

The first-order term can be discretized using the forward formula as follows

The discretized diffusion equation can now be written as

Assuming that the equation can be rearranged to become

Where . This can be easily solved using an explicit scheme. An alternative discretization that can be used is the ADI scheme. Using the ADI scheme the discretization becomes

This is a two step implicit method. In the first step the x-discretization is set to be implicit and the y-discretization is set to be explicit. The order is reversed during the second step. This allows for the use of two tri-diagonal matrices instead of one pentadiagonal matrix. The equations above simplify to

The left hand side of the equation is solved implicitly while the right hand side is solve explicitly.

**Computer Specifications:**

**Model:** Macbook Pro (15-inch, 2018)  
**CPU:** Core i7-8850H - Intel

**Number of Cores:** 6

**Number of Threads:** 12

**Max CPU Clock Speed:** 2.6 GHz

**L1 Cache:** 32 KB

**L2 Cache:** 256 KB (Per core)

**L3 Cache:** 9 MB

**DRAM Size:** 16 GB 2400 MHz DDR4